

A volume in the *Discrete Mathematics and Its Applications* series Edited by Kenneth H. Rosen, AT&T Laboratory, Middletown, New Jersey, USA

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