

A volume in the Discrete Mathematics and Its Applications series
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## A thorough presentation of number theory

Sums of Squares of Integers covers topics in combinatorial number theory as they relate to counting representations of integers as sums of a certain number of squares. The book introduces a stimulating area of number theory where research continues to proliferate. It is a book of "firsts" - namely it is the first book to combine Liouville's elementary methods with the analytic methods of
 modular functions to study the representation of integers as sums of squares. It is the first book to tell how to compute the number of representations of an integer $n$ as the sum of $s$ squares of integers for any $s$ and $n$. It is also the first book to give a proof of Szemerédi's theorem, and is the first number theory book to discuss how the modern theory of modular forms complements and clarifies the classical fundamental results about sums of squares.

The book presents several long-standing, but still interesting and instructive, examples of modular forms. Two chapters develop useful properties of the Bernoulli numbers and illustrate arithmetic progressions, proving the theorems of van der Waerden, Roth, and Szemerédi. The book also explains applications of the theory to three problems that lie outside of number theory in the areas of cryptanalysis, microwave radiation, and diamond cutting. The text is complemented by the inclusion of over one hundred exercises to test the reader's understanding.

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## Features

- Contains over 100 exercises to facilitate comprehension
- Presents a proof of Szemerédi's theorem
- Discusses how the modern theory of modular forms complements the classical fundamental results
- Includes applications to fields such as cryptography, crystallography, and thermal radiation


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